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THE $\mathbf{N-\Delta}$ WEAK AXIAL-VECTOR AMPLITUDE $\mathbf{C_5^A(0)}$

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Abstract

The weak $N-\Delta$ axial-vector transition amplitude $\langle\Delta|\,A^{\mu}_{\pi^+}\,|N\rangle$ —important in N^* production processes in general and in isobar models describing $\nu_\mu N \to \mu\Delta$ processes in particular —is examined using a broken symmetry algebraic approach to QCD which involves the realization of chiral current algebras. We calculate a value for the form factor $C_5^A(0)$ in good agreement with experiment.

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I. INTRODUCTION

The $\mathbf{N}-\mathbf{\Delta}$ weak axial-vector transition matrix element is important when one considers: Neutrino quasielastic scattering $(\nu_\mu + n \to \mu^- + p)$; Δ^{++} production reactions $(\nu_\mu + p \to \mu^- + \Delta^{++})$; Hyperon semi-leptonic decays; Increased understanding of higher symmetries and relativistic and non-relativistic quark models; Models involving isobars; Dynamical calculations involving QCD; Dispersion relations; and Current algebras [1,2].

II. THEORY: THE Δ^{++} PRODUCTION PROCESS

The Δ^{++} production process can be studied in our model by considering the low energy matrix element

$$\langle \mu^- \Delta^{++} | \nu p \rangle = \tfrac{G}{\sqrt{2}} J^h_\mu J^\mu_l = \tfrac{G}{\sqrt{2}} \langle \Delta^{++} | V_\mu - A_\mu | p \rangle J^\mu_l ,$$

where $J^h_\mu \equiv$ hadronic weak current and $J^\mu_l \equiv$ leptonic current.

V_μ (A_μ) is the hadronic vector (axial) current.

With our normalization, [1] nucleon–nucleon hadronic matrix elements may be written as:

$$\langle B_2(p_2, \lambda_2) | J^h_\mu | B_1(p_1, \lambda_1) \rangle = \frac{1}{(2\pi)^3} \sqrt{\frac{m_1 m_2}{E_1 E_2}} \bar{u}_2(p_2, \lambda_2) \left[\Gamma_\mu \right] u_1 (p_1, \lambda_1) \tag{1}$$

where

$$\begin{aligned} \Gamma_\mu \; &= \; f_1(q''^2)\gamma_\mu + (if_2(q''^2)/m_1)\sigma_{\mu\nu} q''^\upsilon + (if_3(q''^2)/m_1) q''_\mu \\ &+ \; \{ g_A(q''^2)\gamma_\mu + (ig_P(q''^2)/m_1)\sigma_{\mu\nu} q''^\upsilon + (ig_3(q''^2)/m_1) q''_\mu \} \gamma_5 \end{aligned} \tag{2}$$

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[1] We normalize physical states according to $< \overrightarrow{p}' \mid \overrightarrow{p} >= \delta^3(\overrightarrow{p}' - \overrightarrow{p})$. Dirac spinors are normalized by $\overline{u(p)}u(p) = 1$. Our conventions for Dirac matrices are $\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$ with $\gamma_5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3$, where $g^{\mu\nu} =$ Diag $[1, -1, -1, -1]$. The Ricci-Levi-Civita tensor is defined by $\varepsilon_{0123} = -\varepsilon^{0123} = 1 = \varepsilon_{123}$.

In Eqs. (1) and (2), $u_1(p_1, \lambda_1)$ is the Dirac spinor for the inital state (octet) baryon which has mass m_1, four momentum p_1, and helicity λ_1. Similarly, $u_2(p_2, \lambda_2)$, is the Dirac spinor for the final state (octet) baryon which has mass m_2, four momentum p_2, and helicity λ_2, and $q'' \equiv p_2 - p_1$.

When the initial baryon is a decuplet state and the final state baryon is a octet state we have (in the notation of Mathews [3]):

$$\langle B_2(p_2,\lambda_2)| J^h_\mu |B'_1(p_1,\lambda_1)\rangle = \frac{1}{(2\pi)^3}\sqrt{\frac{m'_1 m_2}{E'_1 E_2}}\bar{u}_2(p_2,\lambda_2)\left[\Gamma_{\mu\beta}\right] u^\beta_1(p_1,\lambda_1) \tag{3}$$

$$\begin{aligned}\Gamma_{\mu\beta} &= (f'_1(q^2) + g'_1(q^2)\gamma_5)g_{\mu\beta} + (f'_2(q^2) + g'_2(q^2)\gamma_5)\gamma_\mu q_\beta \\ &+ (f'_3(q^2) + g'_3(q^2)\gamma_5)q_\mu q_\beta + (f'_4(q^2) + g'_4(q^2)\gamma_5)p_{1\mu} q_\beta\end{aligned} \tag{4}$$

Where $u^\beta_1(p_1, \lambda_1)$ is a Rarita-Schwinger spinor and where the f'_i and g'_i are axial-vector and vector form factors respectively. (Or in the notation of C. H. Llewellyn-Smith [1] when we write only the axial-vector part, we obtain):

$$\begin{aligned}\Gamma^{Axial}_{\mu\beta} &= C^A_5(q^2)g_{\mu\beta} \\ &+ C^A_6(q^2)q_\mu q_\beta/m^2 \\ &+ C^A_4(q^2)\left\{(p_1\cdot q/m^2)g_{\mu\beta} + q_\beta(p_1+p_2)_\mu/(2m^2) + q_\mu q_\beta/(2m^2)\right\} \\ &+ C^A_3(q^2)\left\{((m^*-m)/m)g_{\mu\beta} + q_\beta\gamma_\mu/m\right\}\end{aligned} \tag{5}$$

The usual Cabibbo assumptions (extended to acknowledge the existence of c, b, and t quarks–*i.e.* one utilizes the Kobayashi and Maskawa (K-M) matrix) are then invoked in order to reduce the number of form factors in Eq.(2) from six to four–namely $f_1(q^2)$, $f_2(q^2)$, $g_1(q^2)$,and $g_P(q^2)$.

These assumptions are:

- Universality of the coupling of the leptonic current to hadronic current;

- J^h_μ components transform like the charged members of the $SU(3)$ $J^P = 0^-$ octet;

- Generalized CVC (isotriplet current hypothesis) holds—implies that f_1 and f_2 can be calculated from the known proton and neutron electromagnetic form factors and that $f_3 = 0$;
- No second class currents exist—implies that $g_3 = 0$.

With those assumptions, Eq.(2) then effectively reduces to:

$$\begin{aligned} \Gamma_\mu \;=\; & f_1(q''^2)\gamma_\mu + (if_2(q''^2)/m_1)\sigma^{\nu}_{\mu\nu}q'' + \\ + \; & g_A(q''^2)\gamma_\mu\gamma_5 + (ig_P(q''^2)/m_1)\sigma_{\mu\nu}q''^{\upsilon}\gamma_5. \end{aligned} \tag{6}$$

For example the well known nucleon weak axial-vector form factor $g_A({q''}^2)$ can be parametrized by

$$g_A({q''}^2)/g_A(0) \cong \left[1 - {q''}^2/m_A^2\right]^{-2},$$

where

$$\langle p, p_2| \, A^{\mu}_{\pi^+}(0) \, |n, p_1\rangle \approx (2\pi)^{-3}\sqrt{(mm_n)/(E_{p_1}E_{p_2})}\bar{u}_p(p_2)\left[g_A({q''}^2)\gamma^\mu\gamma_5\right]u_n(p_1),$$

$m_n =$ neutron mass, and ${q''}^2 = (p_2 - p_1)^2$.

III. PREVIOUS RESULTS AND METHODOLOGY

We consider helicity states with $\lambda = +1/2$ (i.e. spin non-flip sum rules) and the non-strange $(S = 0)$ $L = 0$ ground state baryons ($J^{PC} = \frac{1}{2}^+, \frac{3}{2}^+$). It is well-known that if one defines the axial-vector matrix elements:

$$\langle p, 1/2| \, A_{\pi^+} \, |n, 1/2\rangle \equiv f = g_A(0),$$

$$\left\langle \Delta^{++}, 1/2\right| A_{\pi^+} \left|\Delta^+, 1/2\right\rangle \equiv -\sqrt{\frac{3}{2}}\, g,$$

$$\left\langle \Delta^{++}, 1/2 \right| A_{\pi^+} \left| p, 1/2 \right\rangle \equiv -\sqrt{6}\, h,$$

and applies asymptotic level realization to the chiral $SU(2) \otimes SU(2)$ charge algebra $[A_{\pi^+}, A_{\pi^-}] = 2V_3$, then

$$h^2 = (4/25)f^{\,2} \ (the\ sign\ of\ h = +(2/5)f) \quad and \qquad g = (-\sqrt{2}/5)f.$$

If one further defines (suppressing the index μ):

$$< \Delta^+, 1/2, \vec{s}\,|\, j_3|\, \Delta^+, 1/2, \vec{t} >\equiv a, \qquad\qquad < p, 1/2, \vec{s}\,|j_3|\, p, 1/2, \vec{t} >\equiv b,$$

$$< n, 1/2, \vec{s}\,|\, j_3|\, \Delta^0, 1/2, \vec{t} >\equiv c, \qquad\qquad < \Delta^0, 1/2, \vec{s}\,|\, j_3|\, n, 1/2, \vec{t} >\equiv d$$

(note that other required matrix elements of j_3 can then be obtained easily from the double commutator $[\,[j_3^\mu(0), V_{\pi^+}]\,, V_{\pi^-}] = 2\, j_3^\mu(0))$
and if one also inserts the algebra $[j_3^\mu(0)$,$A_{\pi^+}] = A^\mu_{\pi^+}(0)$ ($j^\mu \equiv j_3^\mu + j_S^\mu$, where $j_3^\mu \equiv$ isovector part of j^μ and j_S^μ is isoscalar) between the ground states $\langle B(\alpha, \lambda = 1/2, \overrightarrow{s})|$and $\left|B'(\alpha, \lambda = 1/2, \overrightarrow{t}\,)\right\rangle$ with $|\overrightarrow{s}| \rightarrow \infty, \left|\overrightarrow{t}\right| \rightarrow \infty$, where $\langle B(\alpha)|$ and $|B'(\beta)\rangle$ are the following $SU_F(2)$ related combinations: $\langle p, n\rangle$, $\langle p, \Delta^0\rangle$, $\langle \Delta^{++}, p\rangle$, $\langle n, \Delta^-\rangle$, $\langle \Delta^{++}, \Delta^+\rangle$, $\langle \Delta^+, \Delta^0\rangle$, $\langle \Delta^0, \Delta^-\rangle$, $\langle \Delta^+, n\rangle$, then one obtains (we use $< N|\, j_S^\mu|\, \Delta >= 0$) the constraint equations (not all independent):

$$2fb - \sqrt{2}h(c+d) = f^{L=0}(\lambda = 1/2) \left\langle p \right| A^\mu_{\pi^+} \left| n \right\rangle, \tag{7}$$

$$\sqrt{2}h(a+b) + (-\sqrt{2}g - f)c = f^{L=0}(\lambda = 1/2) \left\langle p \right| A^\mu_{\pi^+} \left| \Delta^0 \right\rangle, \tag{8}$$

$$\sqrt{6}h(-3a+b) + \sqrt{3/2}gd = f^{L=0}(\lambda = 1/2) \left\langle \Delta^{++} \right| A^\mu_{\pi^+} \left| p \right\rangle, \tag{9}$$

$$\sqrt{6}h(3a-b) - \sqrt{3/2}gc = f^{L=0}(\lambda = 1/2) \left\langle n \right| A^\mu_{\pi^+} \left| \Delta^- \right\rangle, \tag{10}$$

$$-\sqrt{6}ga + \sqrt{6}hc = f^{L=0}(\lambda = 1/2) \left\langle \Delta^{++} \right| A^\mu_{\pi^+} \left| \Delta^+ \right\rangle, \tag{11}$$

$$-2\sqrt{2}ga+\sqrt{2}h(c+d)=f^{L=0}(\lambda=1/2)\left\langle \Delta^{+}\right| A^{\mu}_{\pi^{+}}\left|\Delta^{0}\right\rangle , \tag{12}$$

$$-\sqrt{6}ga+\sqrt{6}hd=f^{L=0}(\lambda=1/2)\left\langle \Delta^{0}\right| A^{\mu}_{\pi^{+}}\left|\Delta^{-}\right\rangle , \tag{13}$$

$$-\sqrt{2}h(a+b)+(f+\sqrt{2}g)d=f^{L=0}(\lambda=1/2)\left\langle \Delta^{+}\right| A^{\mu}_{\pi^{+}}\left|n\right\rangle . \tag{14}$$

Applying asymptotic level symmetry, Eqs. (7)–(14) immediately imply that

$$d=c \tag{15}$$

and

$$a=b+[-\frac{1}{4}\frac{g}{h}-\frac{1}{2\sqrt{2}}\frac{f}{h}]c. \tag{16}$$

One can calculate $f^{L=0}(\lambda=1/2)$ easily by setting $\mu=0$, restoring the x dependence to the matrix elements and integrating over $d\vec{x}$.

We find that $f^{L=0}(\lambda = 1/2) = 1$. Eqs. (7)–(14) then relate the weak matrix elements $\langle\Delta^{+},1/2,\vec{s}|\, A^{\mu}_{\pi^{+}}(0)\left|\Delta^{0},1/2,\vec{t}\right\rangle$, $\langle\Delta^{+},1/2,\vec{s}|\, A^{\mu}_{\pi^{+}}(0)\left|n,1/2,\vec{t}\right\rangle$, and $\langle p,1/2,\vec{s}|\, A^{\mu}_{\pi^{+}}(0)\left|\Delta^{0},1/2,\vec{t}\right\rangle$ to the matrix elements $\left\langle p,1/2,\vec{s}\right|j^{\mu}_{3}(0)\left| p,1/2,\vec{t}\right\rangle$ and $\langle p,1/2,\vec{s}|\, A^{\mu}_{\pi^{+}}(0)\left|n,1/2,\vec{t}\right\rangle$.

In fact, one discovers the important relation

$$(8h-2f)b=2\sqrt{2}\left\langle p\right| A^{\mu}_{\pi^{+}}\left|\Delta^{0}\right\rangle-\left\langle p\right| A^{\mu}_{\pi^{+}}\left|n\right\rangle . \tag{17}$$

IV. RESULTS

We now choose $\mu=0$ and take the limit $|\overrightarrow{s}|\to\infty$ and $\left|\overrightarrow{t}\right|\to\infty$ ($\overrightarrow{s}$ and $\overrightarrow{t}$ are both taken along the z-axis), then $q^{2}=q^{''2}=\widetilde{q}^{2}=0$. We find that

$$\begin{aligned}(2\pi)^{3}<p,1/2,\vec{s}|j^{0}_{3}|\,p,1/2,\vec{t}>\to&\ [1-\widetilde{q}^{2}/(4m^{2})]^{-1}\,G^{V}_{E}(\widetilde{q}^{2})\\(2\pi)^{3}\left\langle p\right| A^{0}_{\pi^{+}}\left|n\right\rangle\to&\ g_{A}(q^{''2})\\(2\pi)^{3}\left\langle p\right| A^{\mu}_{\pi^{+}}\left|\Delta^{0}\right\rangle\to&\ \sqrt{\tfrac{2}{3}}\tfrac{(m+m^{*})}{2m^{*}}C^{A}_{5}(q^{2})\end{aligned} \tag{18}$$

Thus, Eq. (17) and Eq. (18) then predict that

$$C_5^A(0) = \frac{4}{5}\sqrt{3}\frac{m^*}{(m+m^*)}g_A(0). \tag{19}$$

Numerically Eq. (19) reads (using $g_A(0) = 1.25$, $m^* = 1.232\ GeV/c^2$)

$$C_5^A(0) = 0.98 \tag{20}$$

This value is consistent with PCAC and yields the value $\Gamma(\Delta) \approx 100$ MeV.

V. CONCLUSIONS AND SOME COMPARISONS WITH OTHER MODELS AND EXPERIMENT

- $SU(6)$ Symmetry Predicts

$$C_5^A(q^2=0)^{n\to\Delta^+} = \frac{2}{5}\sqrt{3}\,(g_A/g_V)^{n\to p}.$$

This gives rise to the following dilemma: Does one use the pure $SU(6)$ result $(g_A/g_V)^{n\to p} = 5/3 \Longrightarrow C_A^5(q^2=0)^{n\to\Delta^+} = 1.15$ or does one use the experimental value of $(g_A/g_V)^{n\to p} = 1.25 \Longrightarrow C_5^A(q^2=0)^{n\to\Delta^+} = 0.87$?

Clearly, the value of $C_5^A(q^2=0)^{n\to\Delta^+}$ that one chooses to use can represent almost a factor of two in the predicted value of $\Gamma(\Delta)$.

- Non-Relativistic Conventional (N and Δ space wave functions are identical) Quark Model

$$C_5^A(q^2=0)^{n\to\Delta^+} = \frac{2}{5}\sqrt{3}\,(g_A)^{n\to p} = 0.87.$$

- Static (Yukawa pion-nucleon coupling) Models

$$C_5^A(q^2=0)^{n\to\Delta^+} = \frac{g_{\Delta^{++}p}}{\sqrt{6}g_N}\,(g_A)^{n\to p} = 1.11.$$

- Adler's Model and the Feynman, Kislinger, Ravndal (FKR) Relativistic Quark Model

The predictions of the Adler and FKR models are quite similar. Both models predict (roughly) the same non-relativistic limits with the q^2 dependence of axial-vector transition amplitudes determined by q^2 dependence of nucleon axial-vector form factors, although the FKR model makes the stronger prediction that the dependence is the same. It is also true that in Adler and FKR models the low q^2 dependence is roughly the same as for the Static model.

- Experiment (CERN, Brookhaven, Argonne: by measuring the axial-vector mass M_A) generally favors the Adler model.

- PCAC

From the process $\upsilon n \to \Delta^+$ ($i.e.$ $\upsilon + n \to \mu^- + \Delta^+$), PCAC predicts that

$$C_5^A(q^2 = 0)^{n \to \Delta^+} = \frac{f_\pi g_{\Delta^+ n \pi^+}}{m} = 1.2,$$

If the Goldberger-Treiman relationship $g_A/g_N = f_\pi/\sqrt{2}m$ is exactly satisfied, then the static model prediction and PCAC predictions are the same.

- We conclude that our broken symmetry algebraic approach to the calculation of $C_5^A(q^2 = 0)^{n \to \Delta^+}$ yields results consistent not only with experiment but also with the widely used Adler model. The broken symmetry approach also correctly gives the Δ width, and resolves mass and wave function degeneracy problems present in many widely-used quark models.

REFERENCES

[1] C. H. Llewellyn-Smith, *Phys. Rep.* **3C**, 264 (1972); E. Amaldi, S. Fubini, and G. Furlan, *Pion Electroproduction* (Springer-Verlag, Berlin, 1979.

[2] D. A. Dicus and J. R. Letaw, *Ann. Phys.* **126**, 32 (1980).

[3] J. Mathews, *Phys. Rev.* **137**, B444, (1964).

[4] Milton D. Slaughter, *Phys. Rev.* **C49**, R2894 (1994).

[5] M. Slaughter and S. Oneda, *Phys. Rev. Lett.* **59**, 1641(1987).

[6] M. Slaughter and S. Oneda, *Phys. Rev.* **D39**, 2062, (1989).

[7] S. Oneda, T. Tanuma, and M. D. Slaughter, *Phys. Lett.* **88B**, 343 (1979).

[8] S. Oneda and K. Terasaki, *Prog. Theor. Phys. Suppl.* **82**, 1 (1985).

[9] S. Oneda and Y. Koide, *Asymptotic Symmetry and Its Implication in Elementary Particle Physics* (World Scientific, Singapore, 1991).